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**Beach Nourishment Techniques. Report 2.  
A Means of Predicting Littoral Sediment  
Transport Seaward of the Breaker Zone**

**Army Engineer Waterways Experiment Station Vicksburg Miss**

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TECHNICAL REPORT

# BEACH NOURISHMENT TECHNIQUES

Part 1

## A MEANS OF PREDICTING LITTORAL SEDIMENT TRANSPORT SEAWARD OF THE BREAKER ZONE

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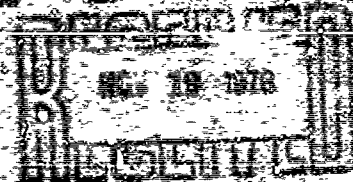
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20. ABSTRACT (Continued).

was Point Pedernales, California. Figures showing the computed and measured longshore sediment transport are included for comparative purposes.

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# PREFACE

The study reported herein was conducted using funds provided by the Operations Division, Office, Chief of Engineers (OCE), under the auspices of the Investigation of Operations and Maintenance Techniques (IOMT) program. Mr. Milt Millard was the OCE Technical Monitor for this work.

The study was conducted during the period August 1973 to April 1975 in the Hydraulics Laboratory of the U. S. Army Engineer Waterways Experiment Station (WES) under the general supervision of Messrs. H. B. Simmons, Chief of the Hydraulics Laboratory; F. A. Herrmann, Jr., Assistant Chief of the Hydraulics Laboratory; Dr. R. W. Whalin, Chief of the Wave Dynamics Division; and Mr. D. D. Davidson, Chief of the Wave Research Branch. The investigation was conducted and this report prepared by Mr. A. W. Garcia, Research Oceanographer, and CPT F. C. Perry, CE.

Directors of WES during this study and the preparation and publication of this report were COL G. H. Hilt, CE, and COL John L. Cannon, CE. Technical Director was Mr. F. R. Brown.

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CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)  
UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

<u>Multiply</u>	<u>By</u>	<u>To Obtain</u>
feet	0.3048	metres
miles (U. S. statute)	1.609344	kilometres
cubic yards	0.7645549	cubic metres
pounds (mass) per cubic foot	16.018546	kilograms per cubic metre
square feet per second	0.09290304	square metres per second
degrees (angle)	0.01745329	radians



## BEACH NOURISHMENT TECHNIQUES

### A MEANS OF PREDICTING LITTORAL SEDIMENT TRANSPORT SEAWARD OF THE BREAKER ZONE

#### PART I: PURPOSE AND SCOPE

1. One means being considered for use in the artificial nourishment of beaches is the artificial deposition of sand seaward of the surf zone. Success of this method requires that the deposited material be placed so that it is entrained into the offshore littoral transport system with a favorable probability of eventually being moved into the surf zone. Previous attempts to accomplish this<sup>1,2</sup> placed the sediment on the basis of grain-size distribution and the expected orbital velocities of the wave regime. A later evaluation<sup>3</sup> of the aforementioned study showed that there was no appreciable movement of the deposited material over a period of four years.

2. In the referenced reports, the "Hjulstrom diagram" was the basis for determining the boundary of eroding versus noneroding velocities for a given sediment size. Although the Hjulstrom diagram is useful, it apparently does not readily lend itself to a viable solution for the problem formulated.

3. The objective of the present study is to develop a mathematical model for obtaining a quantitative estimate of the amount of sediment transported by longshore currents and a qualitative estimate of the amount of sediment transported into the surf zone. The physical processes governing sediment movement within the surf zone are not well understood which necessitates abandoning the search for a quantitative estimate of sediment transport into the surf zone without additional laboratory and field experiments. However, in an approximate fashion, some conclusions can be reached regarding the effects of sediment transported into the surf zone. The most favorable situation (from the viewpoint of beach nourishment) would be that all artificially placed sediment move through the surf zone and deposit on the beach. In practice,

only a fraction of the artificially deposited material will be transported into the surf zone. Even if this quantity does not directly nourish the beach, at the very least it will help saturate the surf zone with sediment and aid (at least temporarily) in retarding erosion of the beach.

4. From the inception of this study it was recognized that the wave data would probably be the least reliable input to the model, and a reliable verification of the model would probably require a comprehensive field testing program. This was pointed out as early as 1952<sup>4</sup> by several investigators who concluded that any study of the movement of sand by waves should be accomplished by direct observations, although at that time the means of doing so had not been sufficiently developed. Sediment tracing techniques have advanced considerably since 1952, and a field verification of the model described in this report and other models under development is being planned. In order to provide at least a tentative order of magnitude verification, a wave hindcast was used in the model to compute the amount of littoral transport at Point Peder-nales, California. The result of this comparison is discussed in Part V.

## PART II: SUMMARY OF PREVIOUS INVESTIGATIONS

### Description of Sediment Movement in the Offshore Zone

5. Most sediment transport produced by gravity waves takes place within the surf zone. Turbulence produced by breaking waves is capable of placing in motion all bottom material except for the largest boulders. It is in the surf zone that the longshore current produced by waves obliquely attacking the shore is most intense. Strong currents combined with extreme agitation can carry a vast quantity of sediment.

6. Seaward of the breaker line, conditions are not as violent, and the ability of gravity waves to transport sediment is considerably reduced. In this region, wave motion can be adequately described by classical small-amplitude wave theory,<sup>5</sup> and the resulting sediment transport can be examined in some detail.

7. Surface gravity waves start to be affected by or "feel" bottom when they enter a depth approximately equal to one-half wavelength. However, it is not until they propagate into much shallower depths that appreciable sediment movement is produced. The depth at which this occurs is difficult to determine due to the large number of variables involved. From laboratory experiments, the relative effects of the various parameters on the initiation, volume, and direction of sediment transport in the offshore zone have been outlined.<sup>6</sup> Numerous field observations indicate depths at which sediment movement is initiated, but all are dependent on local conditions. Occasional sand movement at depths of 100 ft\* off La Jolla, California, is reported in Reference 7. Observations at Point Conception and Arguello, California, revealed that little or no bottom sediment movement occurred at depths greater than 60 ft and only occasional movement in depths between 30 and 60 ft.<sup>8</sup> Very precise measurements of changes in sand levels on the shelf at La Jolla, California, revealed that depth changes of 0.15 ft occurred at a depth of 70 ft.<sup>9</sup> The occurrence of ripples generated by oscillatory wave action at depths

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\* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 3.

of 150 ft off La Jolla, California, was observed by scuba divers and reported by Inman.<sup>10</sup>

8. The mode of sediment movement in the offshore zone is considerably different from the surf zone. Early work by the Beach Erosion Board<sup>11</sup> (now the Coastal Engineering Research Center) shows that only in the surf zone is there an appreciable amount of sediment movement by suspension. In the offshore zone, once movement is initiated, sand moves almost entirely as bed load.

9. The direction of sediment transport in the offshore zone is considered to be generally landward. In 98 percent of the laboratory experiments in Reference 6, transport was observed in the landward direction. This coincides with results predicted by mass transport equations developed by Longuet-Higgins.<sup>12</sup> In general, these equations predict that under the influence of gravity waves there are thin layers of water which move quickly landward at the surface and bottom with a slowed seaward flow of water at intermediate depths. Since sediment transport in the offshore zone is almost all bed load, its corresponding movement will be in the shoreward direction.

10. Even though there is a general landward movement of sediment in the offshore zone, this movement is significantly affected by the longshore current. Although described as being primarily confined to the surf zone,<sup>13,14</sup> the works of Longuet-Higgins and Steward<sup>15</sup> and Thornton<sup>16</sup> show that a significant longshore current can exist in the offshore zone. Thornton<sup>16</sup> verified this by actual field measurements of longshore velocities and sediment transport in the offshore zone.

11. In general, sediment movement by gravity waves in the offshore zone can be described as being initiated in relatively shallow water and moving primarily as bed load diagonally to the shoreline under the influence of both mass transport and longshore currents.

#### Description of Theories on the Movement of Sediment Normal to the Shoreline

12. Present theories on the motion of sediment normal to the shoreline have evolved from the null-point theory, first proposed by

Cornaglia,<sup>17</sup> which states that there are two opposing forces being applied to sediment on the bottom in the offshore zone. These are the onshore wave drift caused by unclosed water particle orbits generated by progressive waves and the offshore component of the gravitational force produced by bottom slope. It is theorized that a point of equilibrium exists between these forces separating onshore and offshore motion for every size, shape, and density of sediment particle. The larger or denser the sediment particle, the closer to shore will be its null point. Shoreward of the null point the particle will move onshore, while seaward from the null point the particle will move offshore until it reaches a point where no motion occurs at all.

13. Many recent investigators have concentrated on the null-point theory. Field studies at La Jolla, California, revealed that sediment tends to decrease in size in the offshore direction.<sup>18</sup> This suggests that sediment is being sorted by the intensity of the forces applied to it and is allowed to establish its point of equilibrium or null point. Laboratory experiments by Ippen and Eagleson<sup>19</sup> and Eagleson, Glenne, and Dracup<sup>20</sup> indicate that although a null point does exist for all sediment sizes under all wave conditions, the net sediment transport at the bed is always in the shoreward direction. Johnson and Eagleson<sup>21</sup> have developed a means of estimating the onshore bed-load transport by using these experimental results. In application of these methods to a section of beach in southern California they were able to predict an average annual onshore transport rate which at least appears reasonable, although it cannot be verified.

14. Other investigators, however, believe that sediment transport in the offshore zone is entirely in the onshore direction due to the inequality of onshore versus offshore wave-induced orbital velocities in the relation of wave advance over a shoaling bottom. Inman<sup>13</sup> indicates that the onshore mass transport of water by waves is continuous and is offset by the occurrence of rip currents that carry the buildup of water in the surf zone offshore again. The sediment transport corresponding to such a circulation system would have sediment moving onshore as bed load under the influence of waves and carried offshore as suspended load

in a rip current where it would be sorted normal to the shore as the rip current loses intensity. Field studies<sup>22,23</sup> discount the effects of the rip current as a return mechanism. These studies indicate that there is general shoreward movement of sediment as bed load along the bottom with the finer material being driven into suspension in the breaker zone and carried offshore at middepths. This also is suggested experimentally by Ippen and Eagleson<sup>19</sup> and Russell and Osorio.<sup>24</sup>

15. Other investigations indicate that the transport of suspended sediment may be more important in the offshore zone than previously suspected, particularly in the process of sediment sorting.<sup>25-31</sup> However, general net transport can still be assumed to be in the shoreward direction as bed load, despite the increased evidence of offshore movement of suspended sediment.

#### Description of Theories on the Movement of Sediment Parallel to the Shoreline

16. Galvin<sup>32</sup> gives a comprehensive review of the theory and data available in North America up to 1967 on longshore current velocities. Nine papers are discussed containing twelve equations of which six can be evaluated using available data. The difficulty in applying the equations discussed by Galvin rests with the fact that they give a mean longshore velocity and one of the necessary requirements for accomplishing the objective of this study is a means of calculating a longshore velocity field. This requirement results from the need to know the depth at which a significant amount of the deposited material can reasonably be expected to be entrained into the littoral current. Gole et al. (personal communication) compared estimates of littoral drift which were computed using three methods, listed below, with dredging data obtained at the Port of Visakhapatnam on the east coast of India.

- a. The Coastal Engineering Research Center's (CERC) formula.
- b. The Putnam-Munk-Traylor formula.
- c. Eagleson's formula.

While the Putnam-Munk-Traylor and Eagleson formulas both gave results that were within 20 percent of the measured drift, the amount computed

using the CERC formula was in excess by a factor greater than four. However, all these methods suffer the same form of deficiency as those discussed by Galvin, i.e., the integrated gross transport is computed.

17. The requirement for determination of the velocity field as a function of distance from the shore (and therefore a function of the water depth) leaves little choice in the basic manner by which the longshore velocity profile is computed. Most studies concerned with this topic<sup>16,33,34</sup> have used what shall be referred to hereinafter as the radiation stress approach, first discussed by Longuet-Higgins and Stewart<sup>15,35</sup> and Bowen.<sup>34</sup> Development of the radiation stress approach will not be recounted here since it is discussed quite eloquently in the above-referenced papers. A detailed discussion of the application of this approach is contained in Part III.

18. Komar and Inman<sup>36</sup> compared two models for determining littoral sand transport with field measurements obtained at Silver Strand Beach, California, and El Moreno Beach, Baja, California. The first model is described as principally intuitive and attempts to correlate the sand transport rate with the longshore component of energy flux per unit length of beach. Komar has shown, however, that Equation 1 can be developed under the basic assumption that the longshore rate of transport of sand is equal to the longshore component of sediment carried forward under the bore of the breaking wave. The second model was proposed by Bagnold<sup>37</sup> and is expressed as

$$I_e = K' (EC_n)_b \cos \alpha_b \langle V_l \rangle U_m \quad (1)$$

where

$I_e$  = rate of transport of immersed weight of sand past section of beach\*

$K'$  = dimensionless factor of proportionality

$E$  = wave energy density

$C_n$  = wave group velocity

---

\* For convenience, symbols and unusual abbreviations are listed and defined in the Notation (Appendix A).

$V_l$  = longshore velocity of unspecified origin

$U_m$  = maximum orbital velocity of waves

$\alpha_b$  = angle of wave crest at breaking with respect to the shoreline.

Komar and Inman concluded that the models are equivalent if driving forces of the longshore current other than those due to wave action from an oblique direction are ignored.



### PART III: OUTLINE AND DISCUSSION OF METHODOLOGY

#### Introduction and Background

19. The basic approach used in this investigation to obtain a quantitative estimate of bed-load sediment transport in the offshore zone was developed over a period of about 25 years by the late Prof. H. A. Einstein and his students at the University of California, Berkeley. Most of this work has been summarized by Prof. Einstein.<sup>4</sup> Einstein's method evolved as an application to the wave-induced sediment transport problem of the Einstein bed-load function,<sup>38</sup> which had originally been developed to calculate sediment transport in streams. Although bed-load movement due to flow in streams is unidirectional, Einstein and his students thought that his theoretical relationships could describe the sediment transport caused by waves. All of their work in developing this approach was conducted as a series of laboratory studies which substantiated the theoretical predictions. The first step was the description of the velocity distribution at the bed due to waves.<sup>39-42</sup> The mechanics of bed-load sediment motion due to waves was then described by Kalkanis<sup>43</sup> and later modified by Abou-Seida.<sup>44</sup> Work on the mechanics of suspended sediment motion also has been undertaken; however, the precise physical description of this process is still an open question and this work was not used in the present study.<sup>45,16</sup>

20. The model used for this investigation will follow the methods of calculating bed-load transport described by Abou-Seida.<sup>44</sup> It is assumed that sediment motion is initiated in the offshore zone by the velocity produced at the bottom by waves. This sediment motion is entirely oscillatory and causes no net movement since small-amplitude wave theory is used. However, since sediment is in motion, it can be acted upon by secondary currents that produce a net transport. These secondary currents, such as longshore and mass transport currents, are not normally intense enough in the offshore zone to initiate sediment motion; however, they can act upon sediment already in motion. In the

present study, these secondary currents are described by the Longuet-Higgins mass transport equation,<sup>12</sup> as originally recommended by Kalkanis,<sup>43</sup> and by the Longuet-Higgins and Stewart radiation-stress analysis of longshore currents.<sup>15</sup>

#### Summary of Assumptions

21. A variety of assumptions and limitations were necessary to develop a practical method for the calculation of bed-load concentration:

- a. The model developed by Kalkanis<sup>43</sup> and Abou-Seida<sup>44</sup> assumes a horizontal bed. This assumption cannot be precisely satisfied in the offshore zone; however, bottom slopes are normally so gradual that the gravitational forces exerted are extremely small compared with those exerted by wave action.
- b. The sediment being moved can be described by its mean particle diameter. In most areas this is a valid assumption since sediment sizes do not vary greatly in the offshore zone, and the standard deviation of most beach sediment samples is small.
- c. Sediment particles are generally spherical in shape. This assumption is satisfactory since beach sediment is primarily quartz grains that tend toward a spherical shape after long exposure to abrasive action. Also, it has been shown experimentally that the shape of sediment particles is much less important than particle size under most conditions of sediment transport.<sup>47</sup>
- d. The model is only applicable to clastic sediment. No attempt was made to include the interparticle forces characteristic of silt- or clay-sized particles. The model may be of questionable accuracy when applied to material in the very fine sand size range (3 to 4  $\phi$  or 0.125- to 0.063-mm diam). Fortunately, most sediment found in the zone of interest is normally sand in the fine-to-medium grain size ranges.
- e. The model as presently designed considers only bed-load sediment movement. Although some sediment is thrown into suspension in the offshore zone, it is not considered to be a large enough quantity or the method of describing it accurate enough to justify an attempt to include it.
- f. Einstein's assumption is used (from his work with river sediment transport<sup>38</sup>) that the bed layer is only two sediment grain diameters thick. Although this assumption

has been criticized by Brenninkmeyer as too restrictive,<sup>48</sup> it is believed that Einstein's assumption is still the best available for practical application at the present time.

- g. Sediment movement occurs only when conditions in the boundary layer can be described as turbulent. Although this assumption is questioned by Komar and Miller,<sup>49</sup> who proposed that movement can occur under laminar flow conditions, the turbulent flow criterion established by Kalkanis<sup>43</sup> is used.

#### Outline of Method Development

22. As previously noted, the model outlined herein is essentially that developed by Kalkanis<sup>43</sup> and modified by Abou-Seida.<sup>44</sup> This brief introduction is only a synopsis of the approach used and the conclusions reached. The original work by Kalkanis or Abou-Seida should be consulted if a more thorough description of the methodology is desired. For a complete description of the development work that led to this model, the summary by Einstein is recommended.<sup>4</sup>

- a. Description of bed motion due to waves. From small-amplitude wave theory (see Wiegel<sup>50</sup> and many others), the horizontal particle velocity due to oscillatory waves at the ocean bottom immediately outside the boundary layer can be described by

$$v = \frac{\pi H}{T \sinh(kh)} \sin(kx - \omega t) \quad (2)$$

where

$v$  = velocity

$H$  = wave height

$T$  = wave period

$k$  = wave number =  $2\pi/L$

$h$  = depth below still-water level

$x$  = distance in the direction of wave propagation

$\omega$  = wave frequency =  $2\pi/T$

$t$  = time

By rearranging terms, this can also be expressed as

$$\begin{aligned}
 v &= \left\{ \left( \frac{2\pi}{T} \right) \left[ \frac{H}{2 \sinh(kh)} \right] \right\} \sin(kx - \omega t) \\
 &= (\omega a) \sin(kx - \omega t) \\
 &= v_o \sin(kx - \omega t) \quad (3)
 \end{aligned}$$

where

$a$  = maximum amplitude of horizontal water displacement at the bottom

$v_o$  = maximum horizontal bottom velocity outside the boundary layer

These two terms ( $a$  and  $v_o$ ) provide a reference from which the velocity distribution within the boundary layer can be calculated. The distribution of velocity within the boundary layer is of the utmost importance because all sand size particles at rest are contained in this thin layer next to the bottom.

b. Velocity distributions within the oscillating boundary layer:

- (1) The velocity distribution within the laminar boundary layer can be described by Stokes classic linearized solution to the Navier-Stokes equations.<sup>51</sup> Using the terminology of Kalkanis, this is expressed as

$$v = v_o \left[ \sin \omega t - e^{-\beta_o z} \sin(\omega t - \beta_o z) \right] \quad (4)$$

where

$$\beta_o = 1/\delta = \sqrt{\omega/2\nu}$$

$\delta$  = boundary layer thickness as defined by Schlichting<sup>52</sup>

$\nu$  = kinematic viscosity

$z$  = distance from the theoretical bed

This solution has been extended to the case of the turbulent boundary layer by Kalkanis, by generalizing the equation for the laminar velocity distribution into the form

$$v = v_o \left\{ \sin \omega t - f_1(z) \sin [\omega t - f_2(z)] \right\} \quad (5)$$

Combining the two sine functions vectorially, the result can be written in the form

$$v = v_0 \left[ 1 - 2f_1(z) \cos f_2(z) + f_1^2(z) \right]^{1/2} \sin (\omega t - \theta) \quad (6)$$

where

$$\theta = \text{phase angle} = \tan^{-1} \left[ \frac{f_1(z) \sin f_2(z)}{1 - f_1(z) \cos f_2(z)} \right]$$

$f_1(z)$  = velocity amplitude function

$f_2(z)$  = phase shift function

Both  $f_1(z)$  and  $f_2(z)$  are experimentally determined functions that take a different form depending on the type roughness of the surface over which the velocity profile is being produced. These empirical functions were derived in laboratory experiments for a smooth bed and with two- and three-dimensional roughness and are summarized by Einstein.<sup>4</sup> For beds composed of either sand or gravel, these functions are expressed by

$$f_1(z) = 0.5e^{-\left(133z/a\beta_0\epsilon\right)}$$

$$f_2(z) = 0.5(\beta y)^{2/3} \quad (7)$$

The roughness diameter ( $\epsilon$ ) is assumed equal to the mean sediment diameter ( $D_{50}$ ). These expressions were obtained from experiments that covered the following ranges of values

$$0.0009 \text{ ft} \leq \epsilon \leq 0.0453 \text{ ft}$$

$$0.104 \text{ ft} \leq a \leq 2.00 \text{ ft}$$

$$0.169 \leq \omega \leq 5.82$$

Water viscosity (i.e. temperature and salinity) was held fixed during the experiments.

(2) As a caution, it must be mentioned that not all

investigators agree that the turbulent boundary layer velocity profile can be described in this manner. In his review of the current state of the art in boundary layer research, Teleki<sup>53,p 52</sup> states that "there is no adequate theory for the mechanics of turbulent flow at this time." More specifically, Noda<sup>54</sup> noted that Kalkanis' empirical description of the turbulent velocity distribution was not verified by his experimental data.

- c. Flow regime transition criteria. Since it has been assumed that sediment will begin to move only under turbulent conditions, criteria must be established to determine what flow characteristics constitute transition from laminar to turbulent conditions. These criteria have been established experimentally from the work of Li,<sup>39</sup> Manohar,<sup>41</sup> and Kalkanis<sup>42</sup> for the three types of bed roughness mentioned previously. For beds composed of sand and gravel, Abou-Seida<sup>44</sup> recommends a Reynolds number criterion of

$$R_c = \frac{Ev}{\nu} = 104 \quad (8)$$

For all Reynolds number values greater than or equal to this criterion, the flow regime is considered turbulent and the empirical velocity distribution previously described is applicable. The criterion for the transition from laminar to turbulent flow conditions is the subject of a continuing controversy. In his review of the subject, Teleki<sup>53</sup> found a wide variation in the Reynolds number criteria that describe this transition and he thought that none were satisfactory. In particular, Sleath criticizes this Reynolds number criterion for being based on insufficient data.<sup>55</sup>

- d. Motion of sediment due to wave motion:

- (1) Once the velocity distribution within the turbulent boundary layer is known, the amount of sediment in motion can be found by use of the bed-load equation derived by Kalkanis<sup>43</sup> and extended by Abou-Seida.<sup>44</sup> This derivation follows the probabilistic procedure developed by Einstein.<sup>38</sup> Since a derivation of this equation is rather lengthy, only the most applicable information will be described here. Einstein provides a short synopsis of the derivation.<sup>4</sup>
- (2) The bed-load equation establishes a relation between two nondimensional parameters that describe the fluid system and the resulting sediment transport. The

first of these, the flow intensity function is given as

$$\psi = \left( \frac{\rho_s - \rho_f}{\rho_f} \right) \left( \frac{g D_{50}}{|\bar{v}_a|^2} \right) \quad (9)$$

where

$\psi$  = flow intensity function

$\rho_s$  = density of the sediment particle

$\rho_f$  = density of the fluid

$g$  = acceleration due to gravity

$D_{50}$  = mean sediment diameter

The velocity  $\bar{v}_a$  is the mean unsteady velocity within the boundary layer given by the equation for the turbulent velocity distribution at a point  $0.35 D_{50}$  above the theoretical bed. The velocity at this distance was determined by Einstein and El Samni<sup>56</sup> as best describing the lift forces on each sediment grain for unidirectional flows. Whether this applies equally as well for oscillatory motion has not yet been determined, but it is a reasonable assumption.

- (3) The second of these nondimensional equations is the bed-load function given by

$$\phi = \left( \frac{q_B}{\gamma_s} \right) \sqrt{\left( \frac{\rho_f}{\rho_s - \rho_f} \right) \left( \frac{1}{g D_{50}^3} \right)} \quad (10)$$

where

$q_B$  = oscillatory bed-load rate

$\gamma_s$  = unit dry weight of bed material

- (4) The relation between the flow intensity function and the bed-load function is given by the bed-load equation (Abou-Seida<sup>44</sup>)

$$\frac{A\phi}{1 + A\phi} = \frac{2}{\pi 2\pi} \left[ \int_0^{\pi/2} \int_0^{\infty} e^{-m^2/2} dm d(\omega t) \right. \\ \left. + \int_0^{\pi/2} \int_{-\infty}^0 e^{-m^2/2} dm d(\omega t) \right] \quad (11)$$

where

$m$  = a random variable that has a normal distribution of zero mean and a standard deviation of one

The terms  $A$ ,  $B$ , and  $\eta_0$  are empirically derived constants which best fit the theoretical curve to the experimental data of Kalkanis and Abou-Seida (see Figure 1). The best fit was obtained with values for these terms of:

$$A = 13.3$$

$$B = 6.0$$

$$\eta_0 = 0.5$$

- (5) The flow intensity function as expressed in the bed-load equation has been modified by the following expression to account for a particular physical phenomenon:

$$\psi_* = \xi \psi \quad (12)$$

The term  $\xi$  is a correction factor introduced by Einstein<sup>38</sup> and commonly known as the "hiding factor." This factor was introduced to account for the fact that under turbulent flow conditions very small sediment particles are partially or completely submerged in the laminar sublayer where they are not affected by the velocity fluctuations in the fully turbulent zone. For the case of oscillatory motion, Abou-Seida has determined this correction experimentally. He has found that the correction is related to the parameter  $D_{50}/1.39 \delta^*$ , where  $\delta^*$  is the thickness of the laminar sublayer. From dimensional considerations and experimental results, Einstein and Li<sup>40</sup> found that the thickness of the laminar sublayer in oscillatory flows can be expressed as

$$\delta^* = \frac{550\nu}{v_o} \quad (13)$$

Using this expression, the parameter  $D_{50}/1.39 \delta^*$  can be computed and the hiding factor can be determined from the plot of Abou-Seida's experimental



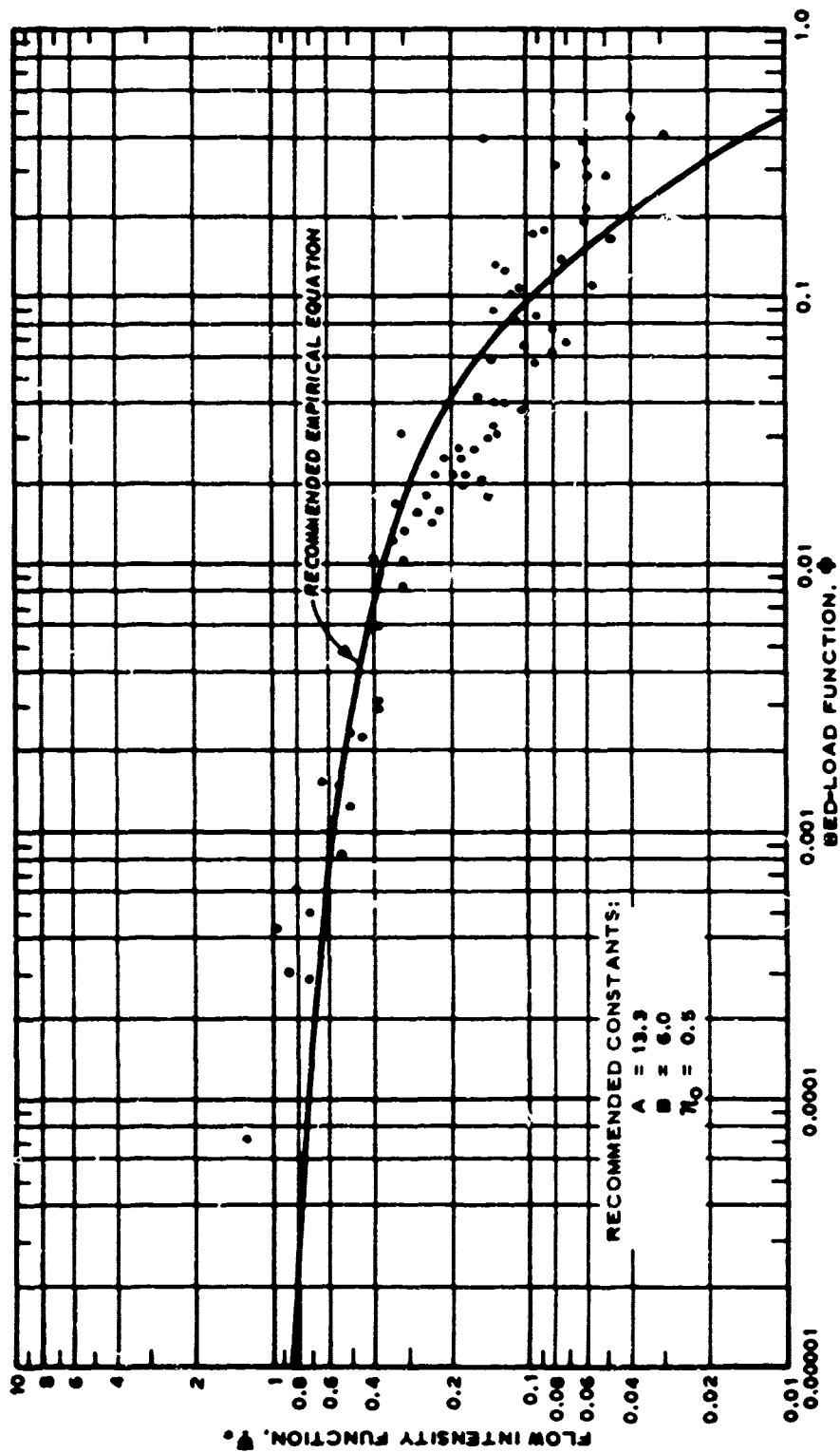


Figure 1. Plot of bed-load equation; data from Kalkanis<sup>43</sup> and Abou-Seida<sup>44</sup>

results (Figure 2) or numerically as will be illustrated later.

- (6) Values of the bed-load function from the corrected flow intensity function can be obtained by use of the curve developed from experimental data (see Figure 1) or by a numerical solution of the bed-load equation that will be described later.
- (7) When a value of the bed-load function is obtained, a value of the oscillatory bed-load rate  $q_B$  also can be established. This represents the average amount of sediment in motion due strictly to the oscillatory wave action;  $q_B$  is expressed in terms of the dry weight of sediment in motion per unit of time per unit width of beach normal to the direction of wave propagation. As such,  $q_B$  represents no net sediment transport.
- (8) Kalkanis<sup>43</sup> obtained a value of net sediment transport in the direction of the secondary current by first reducing  $q_B$  to an average bed-load concentration by developing the following expression

$$C_B = M \frac{q_B}{2D_{50}\bar{v}_m} \quad (14)$$

where

$C_B$  = average bed-load concentration

$M$  = coefficient of proportionality

$2D_{50}$  = thickness of the bed layer (Einstein<sup>38</sup>)

$\bar{v}_m$  = average velocity at a distance  $D_{50}$  from the theoretical bed

In this expression, the average velocity is calculated from the equation for the turbulent velocity distribution at a distance of  $D_{50}$  from the theoretical bed. The coefficient  $M$  has been theoretically evaluated by Kalkanis for small-amplitude waves to have a value of 0.618. Abou-Seida found, however, that for steep waves with short periods and wavelengths, the value of  $M$  must be larger to account for the fact that acceleration effects ignored in Einstein's original development of the bed-load equation can no longer be neglected. At first, it may not seem logical that a correction for acceleration effects should be applied to the resulting sediment transport rather than to the flow intensity function. However, as Einstein has pointed out,<sup>4</sup> the

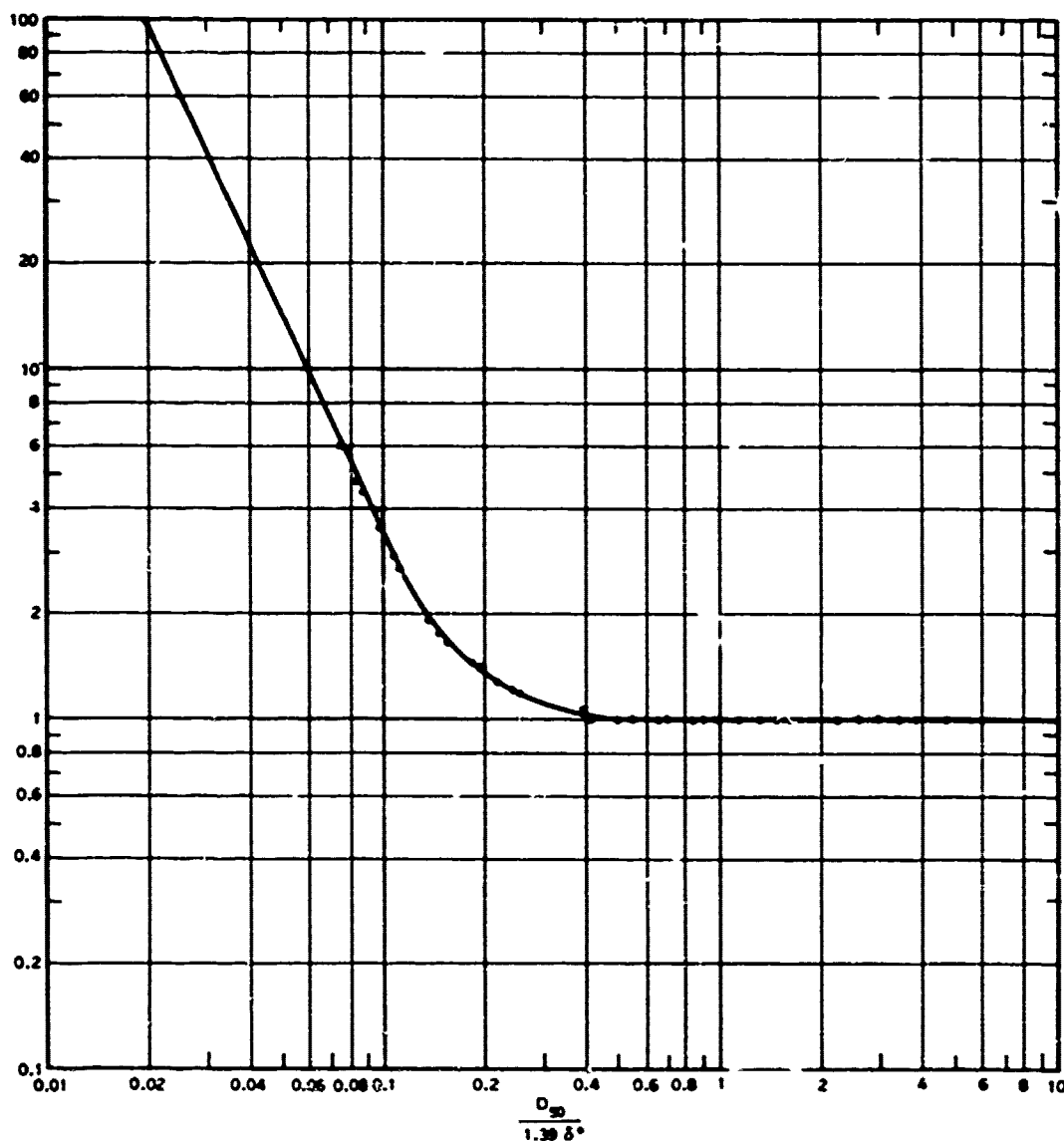


Figure 2. Plot of hiding factor,  $\xi$  ; data from Abou-Seida<sup>44</sup>

acceleration force applies not only to the fixed sediment particles on the bed but to those in motion as well.

- (9) Using experimental data of his own in addition to those of Vincent,<sup>57</sup> Abou-Seida plotted values of  $M$  against the parameter  $a/D_{50}$ , which he concluded was an appropriate indicator of the acceleration effects (Figure 3). This shows that for values of  $a/D_{50}$  less than about 300, the coefficient  $M$  increases sharply as an indication of increased acceleration effects. For values of  $a/D_{50}$  greater than 300, values of  $M$  remain remarkably constant at about the theoretical value of 0.618. For most conditions in the offshore zone where small amplitude theory applies, the theoretical value is satisfactory since acceleration effects may be neglected.

e. Net bed-load sediment transport:

- (1) The average concentration of bed-load material ( $C_B$ ) due to oscillatory wave action provides a measure of the amount of sediment in motion. This is not a measure of sediment transport since it is due to the periodic wave motion that develops no net movement (within the approximations of the Airy wave theory). However, since this material is already in motion, it can be moved readily by any translatory current. Such currents can be tidal, or, as is more prevalent in the offshore zone, currents developed by wave action such as the mass transport and longshore currents. By superposing such a current on the average bed-load concentration, the resulting net sediment transport can be expressed by the following equation

$$Q_B = C_B \int_0^{2D_{50}} U(z) dz \quad (15)$$

where

$Q_B$  = net bed-load sediment transport

$U(z)$  = depth-dependent velocity function

This equation gives a net sediment transport in the direction of the velocity vector.  $Q_B$  is expressed as the dry weight of sediment transported per unit time per unit length normal to the direction of the velocity vector.

- (2) Kalkanis originally recommended that the mass transport current as it is described by Longuet-Higgins<sup>12</sup>

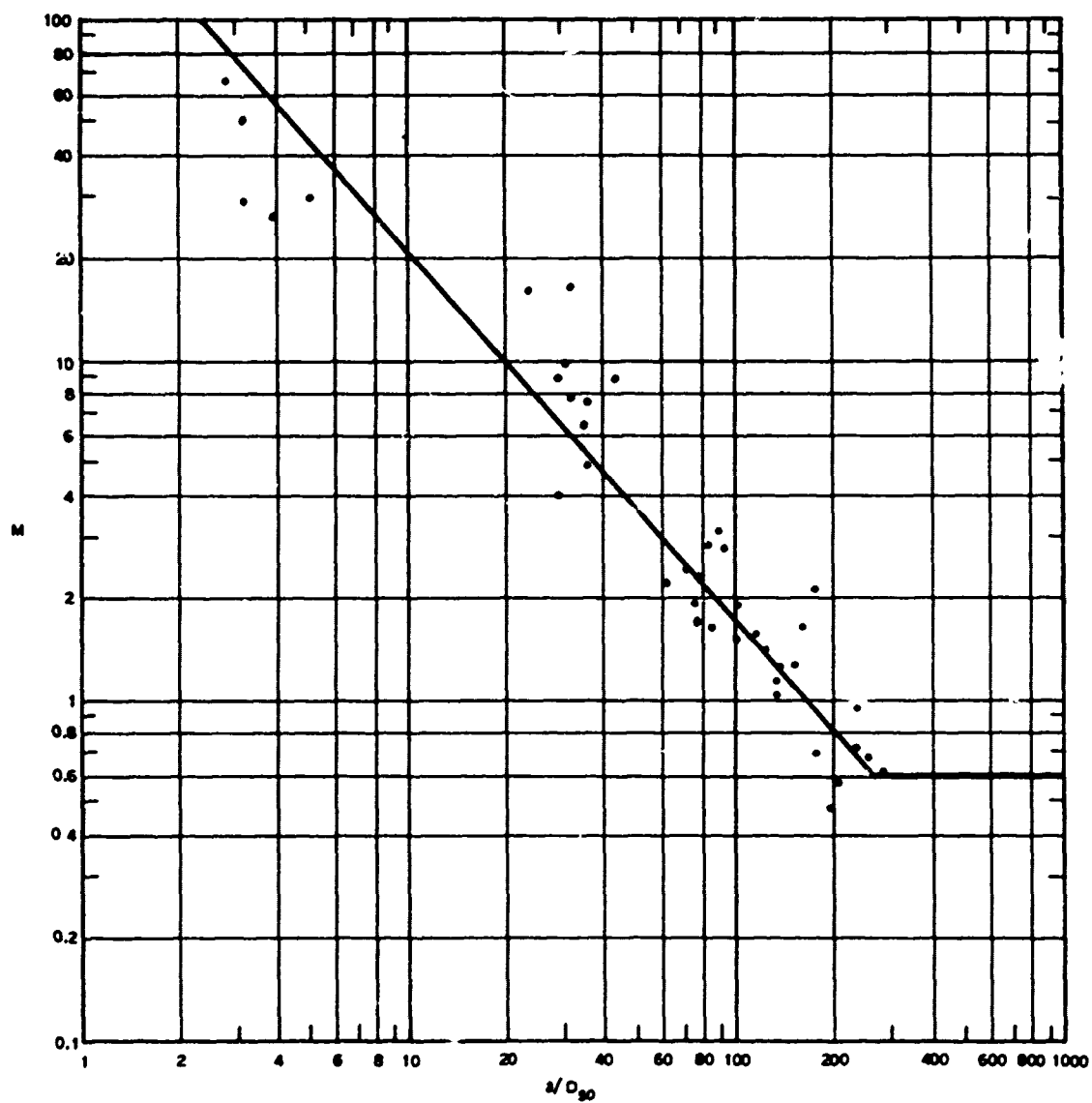


Figure 3. Plot of correction factor for acceleration effects;  
data from Vincent<sup>57</sup> and Abou-Seida<sup>44</sup>

be applied as the translatory current to produce net sediment transport. As a matter of interest, it was from this application of the mass transport velocity to his laboratory experiments that Abou-Seida developed most of the correction factors to the bed-load equation. By simply modifying the notation of the Longuet-Higgins equation, Kalkanis expressed the mass transport velocity as

$$U(z) = \frac{H^2 \omega k}{16 \sinh^2 kh} \left( 5 - 8e^{-\beta_0 z} \cos \beta z + 3e^{-2\beta_0 z} \right) \quad (16)$$

Net sediment transport in the direction of wave propagation can be calculated by using this expression in conjunction with the equation for net transport.

#### Description of Longshore Velocity Field

23. The longshore velocity field is calculated from that described by Longuet-Higgins.<sup>58</sup> The assumptions inherent in this calculation are:

- a. The longshore velocity field is stationary.
- b. The beach is planar and of infinite length.
- c. Velocity is independent of depth in the y direction but return flow is allowed in the x direction.
- d. Currents due to other than wave action (e.g., wind-driven currents) are sufficiently small that their interaction with the wave regime is inconsequential.
- e. Coriolis force is neglected.
- f. Sediment size is uniform over the beach.
- g. Beach slope is sufficiently small so that the effects of wave reflection are negligible.

Assumption (3) does not permit any return flow in the form of rip currents. Evidence indicates that the return of the mass flow due to wave advance takes place at intermediate depths. This has been discussed in detail on a theoretical basis by Longuet-Higgins.<sup>59</sup>

24. No attempt was made during this study to directly verify the validity of the radiation stress approach. Bowen conducted extensive laboratory experiments and found good correlation between the predicted and observed values of the water-surface setdown over the range of wave heights and beach slopes tested.<sup>60</sup> This agreement presumably infers

that confidence can be placed in the velocity field obtained by this method.

25. The basic equations used in this development are those which appear in References 16 and 58. The equations of motion used in the radiation stress approach are (see Figure 4 for definition of coordinate system)

$$\tau_y + \frac{\partial}{\partial x} \left( \mu_e h \frac{\partial v}{\partial x} \right) - \langle B_y \rangle = 0 \quad (17)$$

where

$\tau_y$  = net stress in  $y$  direction

$\mu_e$  = coefficient of horizontal eddy viscosity

$\langle B_y \rangle$  = mean stress on bottom

and, in shallow water

$$\tau_y = \begin{cases} \frac{5}{4} \beta^2 \rho_f (gh)^{3/2} S \left( \frac{\sin \alpha}{c} \right), & x < x_b \\ 0, & x > x_b \end{cases} \quad (18)$$

and

$$\langle B_y \rangle = \frac{2}{\pi} \beta S \rho_f (gh)^{1/2} v \quad (19)$$

where  $S$  is a friction coefficient (see paragraph 26). The horizontal eddy viscosity is calculated from

$$\mu_e = N \rho_f x (gh)^{1/2} \quad (20)$$

where  $N$  is a coefficient of proportionality. From the expected maximum turbulent velocities and maximum mixing length Longuet-Higgins reasons that

$$0 < N < 0.016$$

Assuming a constant beach slope

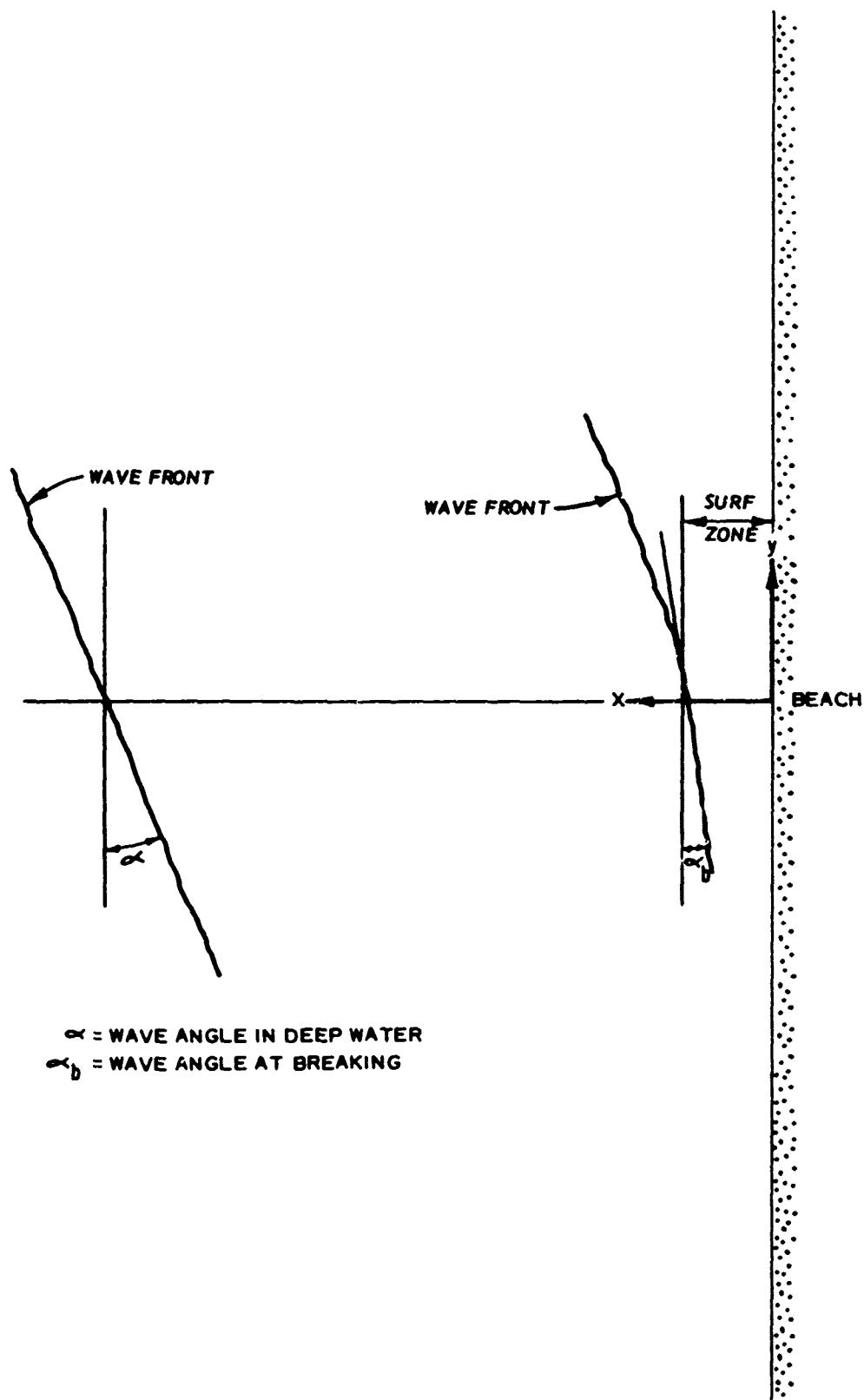


Figure 4. Coordinate system and illustration of wave angles



$$h = \frac{s}{x} \quad (21)$$

where  $s = db/dx$ . Equation 17 can then be rewritten as

$$p \frac{\partial}{\partial x} \left( x^{5/2} \frac{\partial v}{\partial x} \right) - qx^{1/2} v = \begin{cases} -rx^{3/2} & , 0 < x < x_b \\ 0 & , x_b < x < \infty \end{cases} \quad (22)$$

where the values of  $p$ ,  $q$ , and  $r$  are given as

$$\begin{aligned} p &= N \rho_f g^{1/2} s^{3/2} \\ q &= \frac{2}{\pi} \beta S \rho_f g^{1/2} s^{1/2} \\ r &= \frac{5}{4} \beta^2 \rho_f g^{3/2} s^{5/2} \frac{\sin \alpha_b}{\sqrt{gh_b}} \end{aligned} \quad (23)$$

The nondimensional variables

$$X \equiv \frac{x}{x_b}, \quad V \equiv \frac{v}{v_0} \quad (24)$$

are introduced where  $v_0$  is defined as

$$v_0 = \frac{5\pi}{8} \frac{\beta}{S} (gh_b)^{1/2} s \sin \alpha_b \quad (25)$$

From Equations 21 and 22

$$P \frac{\partial}{\partial X} \left( X^{5/2} \frac{\partial V}{\partial X} \right) - X^{1/2} V = \begin{cases} -X^{3/2} & , 0 < X < 1 \\ 0 & , 1 < X < \infty \end{cases} \quad (26)$$

where

$$P = \left( \frac{\pi}{2} \right) \left( \frac{sN}{\beta S} \right) \quad (27)$$

$P$  is a measure of the horizontal mixing. The boundary conditions imposed upon  $V$  at the breaker line ( $X = 1$ ) are that both  $V$  and

$\partial V / \partial X$  are continuous. The complete solution to Equation 26 is

$$V = \begin{cases} B_1 X^{p_1} + AX, & 0 < X < 1 \\ B_2 X^{p_2}, & 1 < X < \infty \end{cases} \quad (28)$$

where the values of  $A$ ,  $p_1$ ,  $p_2$ ,  $B_1$ , and  $B_2$  are given by

$$A = \frac{1}{\left(1 - \frac{5}{2P}\right)}, \quad P \neq \frac{5}{2} \quad (29)$$

$$p_1 = -\frac{3}{4} + \left(\frac{9}{16} + \frac{1}{P}\right)^{1/2}$$

$$p_2 = -\frac{3}{4} - \left(\frac{9}{16} + \frac{1}{P}\right)^{1/2} \quad (30)$$

$$B_1 = \frac{p_2 - 1}{p_1 - p_2} A \quad B_2 = \frac{p_1 - 1}{p_1 - p_2} A \quad (31)$$

The nondimensional form of the velocity profile is obtained by substituting Equations 27, 29, 30, and 31 into Equation 28. Equations 24 and 25 are used to relate results of Equation 28 to the prototype problem. Figure 5 shows a family of current profiles as a function of the mixing parameter  $P$ . This figure illustrates the relative insensitivity of the velocity profiles to the value of  $P$  ( $V$  varies less than an order of magnitude with values of  $P$  over five orders of magnitude).

26. The calculation of typical values of the necessary parameters is illustrated as follows. Several preliminary computations were made using values of  $N$  such that  $0.005 < N < 0.15$ . Results of these calculations were examined, and it was concluded that the system is relatively insensitive to the values of  $N$  within the indicated range.  $N$  was assigned the value of 0.01 for the computations made for this study. An order of magnitude estimate of the Reynolds number was needed to determine a value of the frictional coefficient  $S$ . Using the values in Equation 45 of Longuet-Higgins<sup>58</sup>

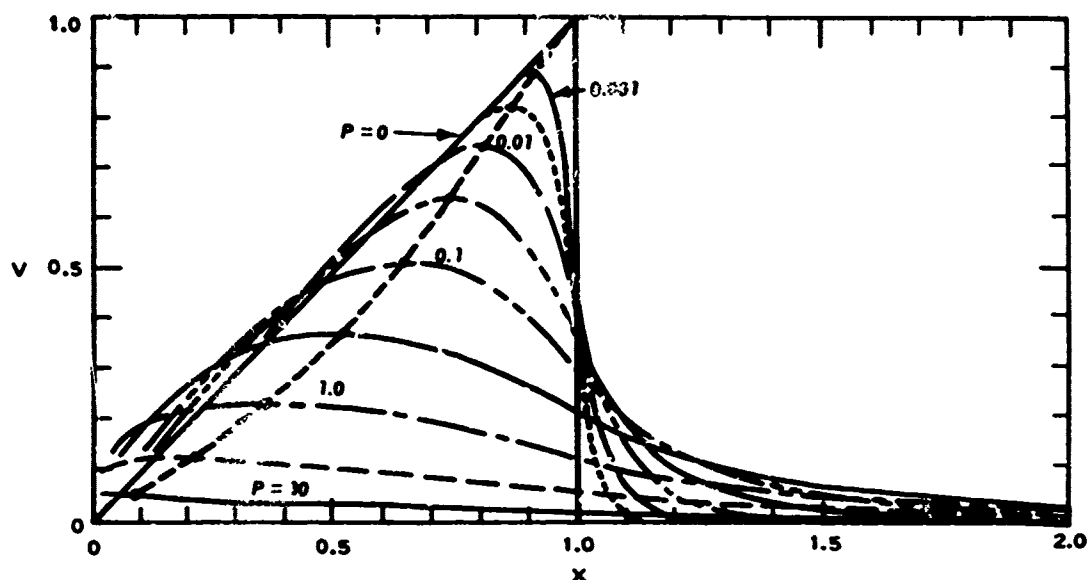


Figure 5. The form of longshore current profiles as given by Equation 28 sequence of values of the mixing parameter  $P$ .  
Adapted from Longuet-Higgins<sup>58</sup>

$$R_e = \frac{\beta^2 gh}{\omega v} \quad (32)$$

where

$$\beta = 0.4$$

$$g = 10 \text{ m/sec}^2$$

$$h = 2 \text{ m}$$

$$\omega = 0.8 \text{ radian/sec}$$

$$v = 1.3 \times 10^{-6} \text{ m}^2/\text{sec}$$

gives the result

$$R_e \approx 3 \times 10^6 \sim O(6) \quad (33)$$

The above values correspond to a wave with a 5-sec period and 2-m breaking height. The second parameter needed is the ratio  $a/\epsilon$  where  $\epsilon$  is a typical value for a roughness element (for our purposes  $\epsilon \approx 0.25 \text{ cm}$ ) and  $a$  is the horizontal excursion of a water particle given by

$$a = \frac{v_{\max}}{\omega}, \quad v_{\max} = \sqrt{gh} \quad (34)$$

thus, for this example,

$$a = \frac{\sqrt{10 \times 2}}{0.8} = 5.6 \text{ m}$$

and

$$\frac{a}{\varepsilon} = \frac{5.6}{0.0025} = 2.2 \times 10^3$$

Reading from Figure 6 (from Prandtl<sup>61</sup>), these values correspond to a

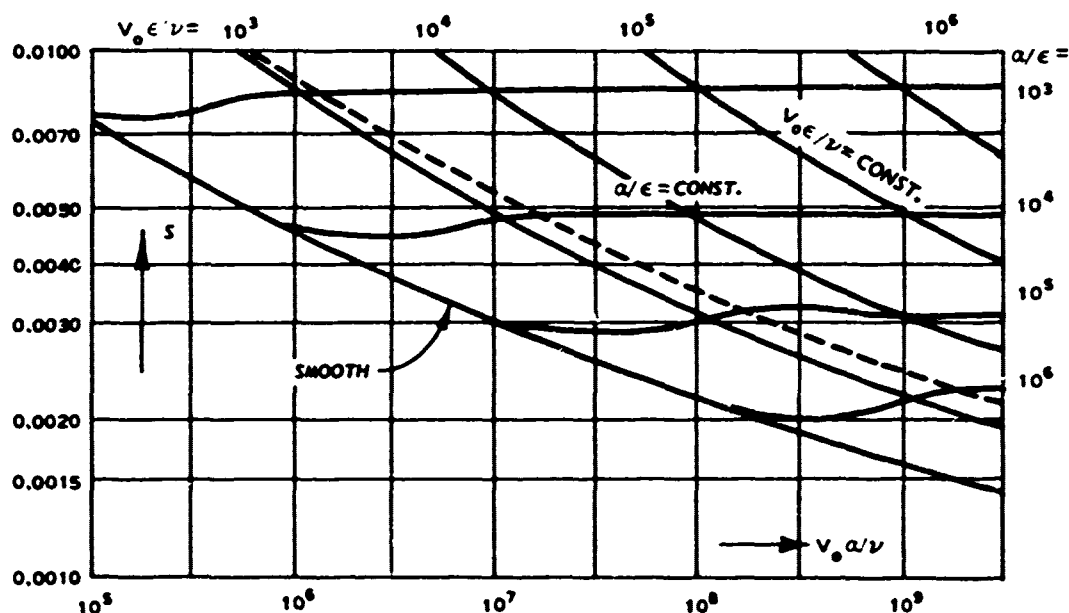


Figure 6. Values of the friction coefficient  $S$  for flow over rough plates (from Prandtl<sup>61</sup>)

friction coefficient  $S \approx 0.008$ . If the parameters  $N$ ,  $S$ , and  $\beta$  are fixed,  $P$  is a function of the bottom slope.

27. To refract and shoal the deepwater waves, the governing equations are those typically used:

$$\text{Snell's law} \quad \frac{\sin \alpha}{\sin \alpha_0} = \frac{v_0}{v} \quad (35)$$

$$\begin{array}{ll} \text{Dispersion} & \frac{\Xi}{\Xi_0} = \tanh \Xi h \\ \text{relation} & \end{array} \quad (36)$$

$$\begin{array}{ll} \text{Conservation of} & \frac{H}{H_0} = \Xi_r \Xi_s \Xi_{fp} \\ \text{energy flux} & \end{array} \quad (37)$$

where  $H$ ,  $\Xi$ , and  $\alpha$  are functions of  $x$ . From Thornton,<sup>16</sup> if percolation is ignored, the energy dissipation due to friction is given by

$$\eta_f = \frac{f_w}{12\pi} \left( \frac{\omega H}{\sinh Kd} \right)^3 \quad (38)$$

where  $f_w$  is obtained from

$$\frac{1}{4\sqrt{f_w}} + \alpha n \frac{1}{\sqrt{f_w}} = -0.08 + \alpha n \frac{\gamma_d}{r} \quad (39)$$

and

$$\gamma_d = \frac{H}{2} \frac{1}{\sinh Kd} \quad (40)$$

Equation 39 is solved by an iterative procedure.

28. To summarize the procedure thus far: the deepwater wave regime is refracted and shoaled into shallow water to the point of breaking, taking account of the effect of dissipation due to bottom friction. Distance from shore, angle of breaking, and breaking wave height are used to compute the longshore current profile, following the procedure of Longuet-Higgins previously outlined. During the numerical shoaling and refraction procedure, the values of water depth, wave height, wavelength, and refraction angle are retained for use in computation of the bed load.

#### PART IV: IMPLEMENTATION

29. To obtain a measure of the distance rate of decrease of velocity within the boundary layer, it was assumed the boundary layer velocity profile is represented by the empirical formula (Schlichting<sup>52</sup>)

$$\frac{U}{U_{\infty}} = \left(\frac{z}{\delta}\right)^{1/n} \quad (41)$$

where  $U_{\infty}$  is the fluid velocity outside the boundary layer and  $n$  is an integer determined by the magnitude of the Reynolds number. For  $R \sim 10^3$  (turbulent),

$$U = U_{\infty} \left(\frac{z}{\delta}\right)^{1/6} \quad (42)$$

where  $\delta$  is thickness of the boundary layer. Because of the difficulty in determining this distance, it is usually taken as the distance above the solid boundary where the velocity reaches 99 percent of the velocity of the well-developed flow exterior to the boundary layer. This latter thickness is denoted by  $\delta^*$  (from Abou-Seida<sup>44</sup>).

$$\delta^* = \frac{1100v}{2U_{\infty}} \quad (43)$$

and the boundary condition

$$U_{\delta^*} = \frac{U_{\infty}}{2} \text{ at } z = \delta^* \quad (44)$$

Combining Equations 42 and 43 and applying the boundary condition

$$\left(\frac{1}{2}\right)^6 = \frac{\left(\frac{1100v}{2U_{\infty}}\right)}{\delta} \quad (45)$$

thence solving for  $\delta$ ,

$$\delta = \frac{2^5 \times 1100v}{U_{\infty}} \quad (46)$$

For a kinematic viscosity of  $1 \times 10^{-5} \text{ ft}^2/\text{sec}$

$$\delta = \frac{0.352}{U_{\infty}} \quad (47)$$

Figure 7 is a plot of the boundary layer thickness as a function of the far-field fluid velocity. For the purposes of our discussion,  $U_{\infty}$  is the longshore current velocity obtained using the radiation-stress approach. The velocity within the boundary layer is designated by  $U_T$  where

$$U_T = U_{\infty} \left( \frac{3}{\delta} \right)^{1/6}, \quad \delta = \frac{0.352}{U_{\infty}} \quad (48)$$

It was decided to try the integrated mean value of  $U_T$ ,  $\bar{U}_T$ , in a first attempt to compute the velocity within the turbulent boundary layer. Schlichting<sup>52</sup> gives

$$\bar{U}_T = 0.79 U_{\infty} \quad (49)$$

for  $n = 6$ . Justification for this can be found by examining Figure 7 where it is evident that  $U_T$  reaches a value of about  $0.7U_{\infty}$  within the first 1/10 of the boundary layer thickness.

30. Three types of information are needed to calculate an average bed-load concentration. These are the characteristic wave parameters (period, wavelength, height, and depth of water), the fluid characteristics (density and viscosity), and the sediment characteristics (density, mean particle diameter, and dry specific weight). The sediment and water characteristics are known initially and wave parameters can be calculated.

31. After the values of a few necessary parameters are obtained ( $a$  and  $u_o$ ), the first step in the computation is to determine if the requirement of turbulent flow conditions at the bed is satisfied. This is simply accomplished by checking to see if the Reynolds number criterion for turbulent conditions is satisfied

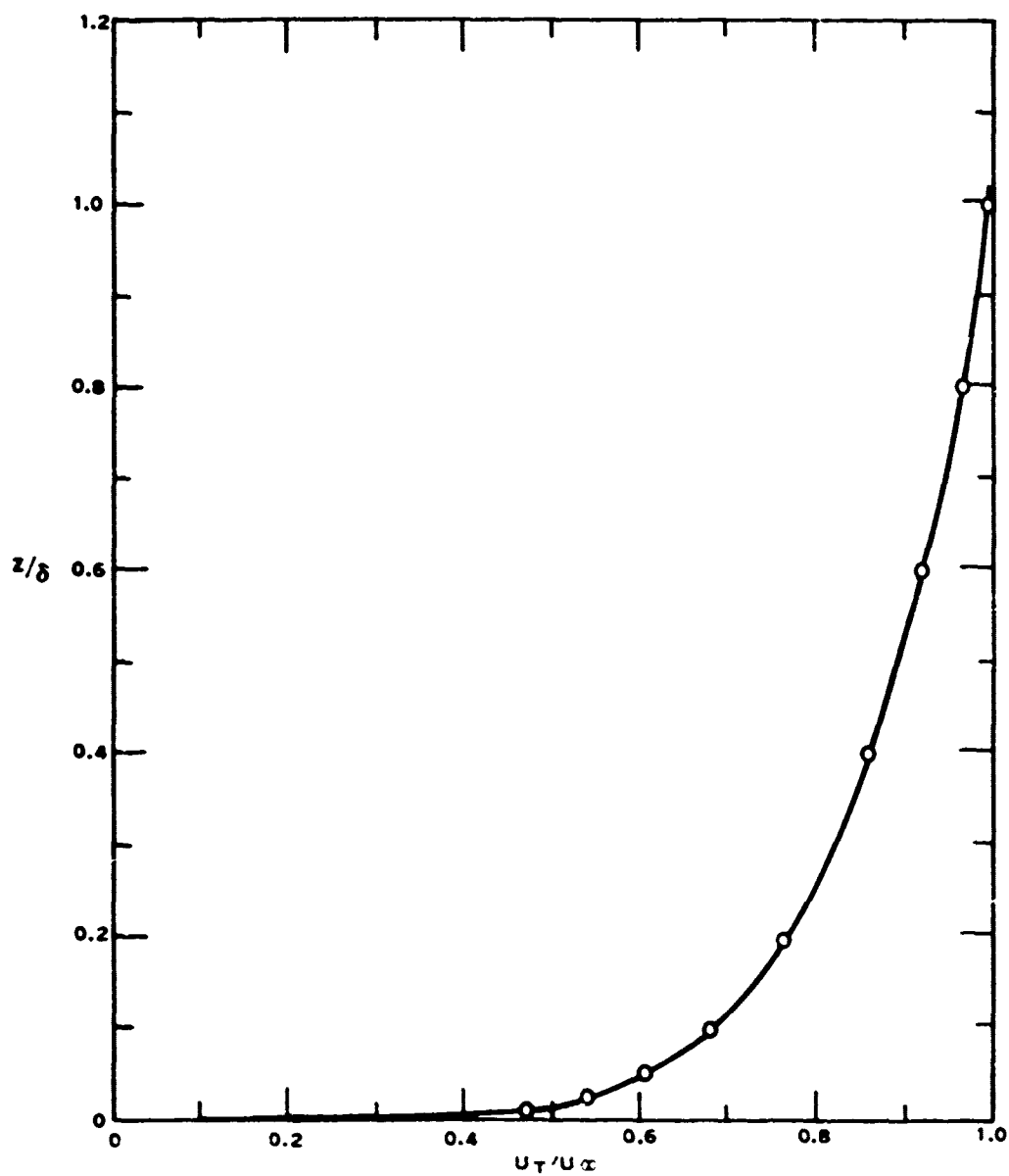


Figure 7. Velocity distribution in boundary layer adjacent to bottom;  $U_\infty$  = longshore velocity



$$\frac{v_o D_{50}}{v} > 104$$

If this requirement is met, the next step is to calculate the average velocity within the boundary layer at a distance of  $0.35 D_{50}$  from the theoretical bed. This is accomplished by using the following modified form of the turbulent velocity distribution equation and the recommended forms of the velocity amplitude and phase shift functions.

$$\begin{aligned} \bar{v}_a &= v_o \left[ 1 - 2f_1(a) \cos f_2(a) + f_1^2(a) \right]^{1/2} \left( \frac{2}{\pi} \right) \\ f_1(a) &= 0.5e^{-(46.55/a\beta_o)} \\ f_2(a) &= 0.248 (\beta_o D_{50})^{2/3} \end{aligned} \quad (50)$$

From this result, the flow intensity function,  $\psi$ , is calculated using Equation 9. The hiding factor  $\xi$  must be determined from a statistical evaluation of Abou-Seida's original data (Figure 2). Upon close examination of these data, four distinct groups of points were observed. A least-squares analysis was performed for each group. This provided the following empirical estimate for the hiding factor:

$$\delta^* = \frac{550.0v}{v_o} \quad (51)$$

$$X = \frac{D_{50}}{1.396 \delta^*} \quad (52)$$

$$\xi = 1.0 \quad \text{for} \quad X \geq 0.4230$$

$$\xi = 0.7315X^{-0.3634} \quad \text{for} \quad 0.4230 > X \geq 0.2173$$

$$\xi = 0.3701X^{-0.8097} \quad \text{for} \quad 0.2173 > X \geq 0.1379$$

$$\xi = 0.0407X^{-1.9241} \quad \text{for} \quad X < 0.1379$$

32. Once the hiding factor is found, the modified flow intensity function is computed using Equation 12 and can then be used to solve the bed-load equation. Abou-Seida<sup>44</sup> integrated the right-hand side of the bed-load equation by using the tables of the normal probability integral in conjunction with

$$\frac{A\phi}{1 + A\phi} = \bar{p}$$

$$\bar{p} = \frac{\sum_{i=1}^N p_i}{N}$$

$$p_i = \frac{1}{\sqrt{2\pi}} \int_{\frac{B\psi_*}{(\cos \phi_i)^2} - \frac{1}{\eta_0}}^{\infty} e^{-m^2/2} dm + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{B\psi_*}{(\cos \phi_i)^2} - \frac{1}{\eta_0}} e^{-m^2/2} dm \quad (53)$$

To evaluate this set of equations, Abou-Seida set  $N$  equal to 9 so that  $\phi_i$  represents the midpoint of one of the nine equal intervals dividing the quarter of the cycle. For the present application, a power series solution is used to compute values of normal probabilities. A value of 90 is used for  $N$  in calculating the bed-load function.

33. The oscillatory bed load is given by

$$q_B = \gamma_s \phi \left[ \left( \frac{\rho_f}{\rho_s - \rho_f} \right) \left( \frac{1}{g D_{50}^3} \right) \right]^{1/2} \quad (54)$$

and the bed-load concentration is calculated from Equation 14.  $\bar{v}_m$  is found by calculating the average velocity within the turbulent boundary layer at a distance  $D_{50}$  above the theoretical bed.  $M$  is determined from a least-squares evaluation of Vincent's<sup>57</sup> and Abou-Seida's<sup>44</sup> original data in conjunction with the theoretical value derived by

Kalkanis.<sup>43</sup> This provides the following formulas for M :

$$Z = \frac{a}{D_{50}} \quad (55)$$

$$M = 0.6180 \quad \text{for } Z > 256.6333$$

$$M = 251.9058Z^{-1.0834} \quad \text{for } Z \leq 256.6333$$

With a value of the bed-load concentration  $C_B$ , net transport can now be calculated for any secondary current using Equation 15. For the mass transport velocity, evaluation of Equation 15 can then be accomplished by a simple integration of Equation 16.

## PART V: MODEL VERIFICATION

34. Verification of any model of coastal processes is extremely difficult due to the scarcity of reliable and quantitative prototype data. In this particular case, information on the onshore transport of sediment is practically nonexistent. However, estimates of the net longshore transport of sediment are numerous and are based on long-term studies that increase their averaged accuracy. It was therefore decided to verify the present model by using it to calculate longshore transport for a section of coast that has a relatively well-established sediment budget together with reasonable estimates of longshore sediment transport. Point Pedernales, California, a rocky promontory located approximately 2 miles north of Point Arguello, was selected for the following reasons:

- a. The immediate offshore bathymetry, characteristics of the bottom sediment, mechanisms of sediment transport in the offshore zone, and sediment budget for this section of coast have been well-documented.<sup>8,60,62</sup>
- b. A deepwater wave hindcast station has been established immediately offshore from this location (34°30'N, 121°00'W) and is based on data for three consecutive years--1956, 1957, and 1958 (National Marine Consultants<sup>63</sup>).
- c. Point Pedernales is a rocky headland that protrudes well into the surf zone, thus effectively blocking any longshore transport of sediment in the surf zone. This creates a situation in which the limitation of the model to the offshore zone does not prohibit its use in determination of the total longshore transport.

### Sources and Quality of Input Data Used

#### Wave data

35. The wave data used for the computation of sediment transport at Point Pedernales are those presented for Station 5, located at 34.5°N, 121.0°W, of the National Marine Consultants.<sup>63</sup> The significant wave heights tabulated in the hindcast were converted to root-mean-square heights by dividing by the factor 1.416 (cf. Longuet-Higgins<sup>59</sup>). It is thought that the root-mean-square wave height was a more accurate

measure of the energy of the wave regime than the significant wave height, and this accuracy would be reflected in the computation of the longshore transport. Since the coastline in this area runs in approximately a straight line from north-northeast to south-southwest, only waves from directions of north to southwest are converging on the study area (Figure 8). Waves (combined sea and swell) from north and southwest higher than 4 ft, the highest listed as 12 ft, occur a combined total of less than one percent of the year and are therefore judged to make an insignificant contribution to the net annual transport. Waves from west-northwest are approximately normal to the coast and although they would produce an onshore transport of sediment, they would not contribute to the total longshore transport. Therefore, only waves from north-northwest, northwest, west, and west-southwest were used in the longshore sediment transport computations.

36. By averaging wave directions into arcs of  $22.5^\circ$ , the hindcast introduces the single most important source of error into the model. The longshore velocity and its associated sediment transport are very sensitive to the angle of wave incidence and the amount of error thus introduced can be significant.

#### Fluid data

37. Basic physical properties of seawater were obtained for nearby Avila Beach, California (National Ocean Survey<sup>64</sup>), from daily observations made at the tide recording station. The following annual averages were used in the computations.

$$\text{Temperature (T)} = 56.8^\circ\text{F} = 13.8^\circ\text{C}$$

$$\text{Density } (\rho_w) = 1.0252$$

$$\text{Salinity (S)} = 33.6 \text{ }^\circ/\text{oo}$$

The value of salinity was converted to chlorinity by use of the empirical relationship recommended by Sverdrup, Johnson, and Fleming.<sup>65, p 51</sup> Although more precise formulas have been developed, this simple relation

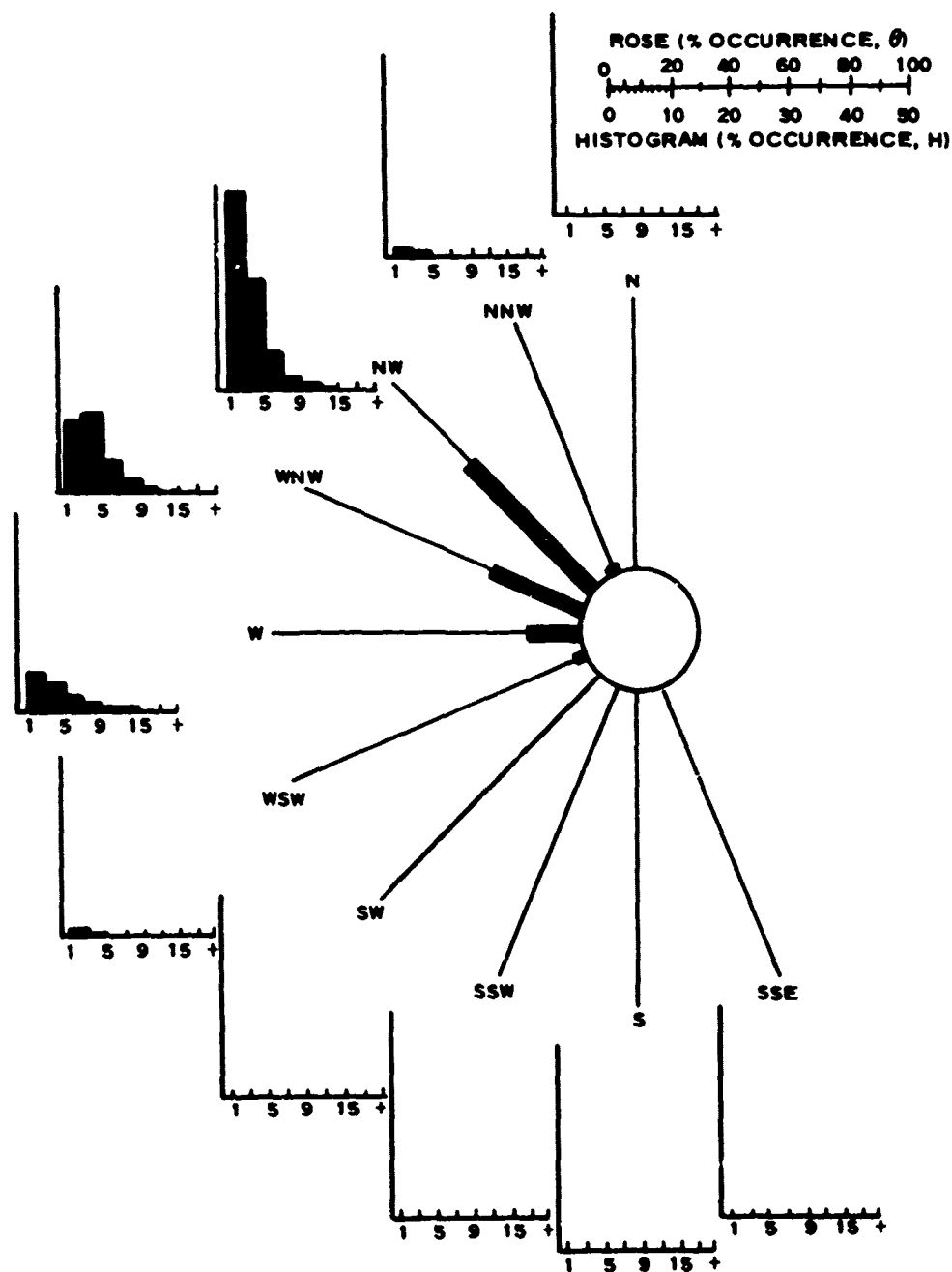


Figure 8. Average annual wave rose for Point Arguello, California (adapted from National Marine Consultants<sup>63</sup>)

is sufficient for the present application. The resulting chlorinity was:

$$\text{Chlorinity (CL)} = 18.6 \text{ ‰}$$

From these data and the relationship developed by Miyake and Koizumi,<sup>66</sup> the dynamic viscosity is:

$$\mu = 1.26(10^{-2}) \text{ poise}$$

Using the established average density, the kinematic viscosity is:

$$\nu = 1.23(10^{-2}) \text{ stokes} = 1.32(10^{-5}) \text{ ft}^2/\text{sec}$$

The value of viscosity used can be a source of minor error since viscosity is sensitive to temperature and also varies with salinity. Errors also can be introduced in areas with spatial variations in temperature, density, and salinity. Only slight variations of these parameters are present in the area under investigation.

38. Preliminary calculations with the model indicate that sediment transport increases with water temperature. This is similar to the trends found by Franco<sup>67</sup> and by Taylor and Vanoni<sup>68</sup> for unidirectional flows. The model also indicates a substantial increase in the depth at which sediment movement is initiated with increasing temperature. This is logical since the increasing water temperature lowers the viscosity, thus reducing the magnitude of the velocity necessary to induce turbulent flow.

#### Bathymetric data

39. One of the assumptions presently contained in the model is that the bathymetry is represented by a simple plane beach. Therefore, the data given by Trask<sup>8</sup> and Bower and Inman,<sup>60</sup> and National Ocean Survey Hydrographic Surveys H-5746, H-5741, and H-5799 were used to develop an average offshore slope of 0.0184. Figure 9 illustrates the slope used in the model as compared with the actual slope off Point Pedernales

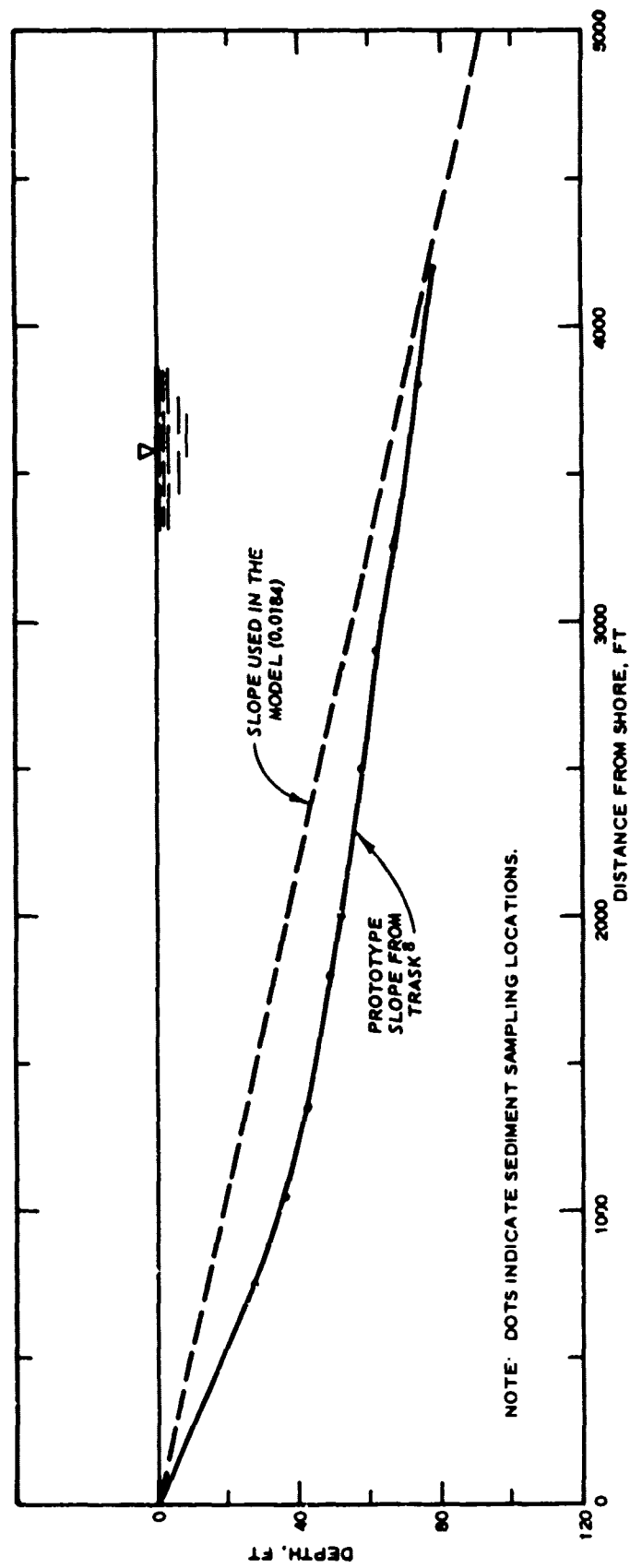


Figure 9. Bottom profile off Point Pedernales, California



in the immediate offshore zone. The use of an averaged slope in this area can be expected to result in a higher computed transport rate than the use of the actual slope because Trask<sup>8</sup> believes all sediment movement in this area takes place in depths shallower than 60 ft.

#### Sediment data

40. The assumption that the model predicts the sediment transport on a beach of a constant uniform grain size coincides well with actual field data available for Point Pedernales. Trask<sup>8</sup> shows that for the entire offshore area shown in Figure 9, the mean grain size of all sediment samples taken varied from only 146 to 153 microns; therefore, an average grain size of 0.15 mm (or  $4.92 \times 10^{-4}$  ft) was used.

41. It was assumed that the sediment was predominantly quartz sand with a density of 2.65. It was also assumed that the dry unit weight of the sediment was 109 pcf as given by Terzaghi and Peck<sup>69</sup> for dense, uniform sand although values as low as 90 pcf can be expected for loose, uniform sand. The larger value was used because saturated fine-grained sand tends to be densely packed.

#### Results of Verification Computations

42. Using the model as presently developed, annual longshore sediment transport volumes were calculated for every applicable combination of intervals of wave period, wave height, and incident angle with all other conditions remaining constant. These values were then totaled for both upcoast and downcoast sediment transport, and the gross and net transport was computed. Two separate computations were conducted in this manner. The first computation considered all sediment transported in the offshore zone (i.e., outside the breaker zone), while the second considered only transport in the offshore zone in depths greater than 12 ft. This latter computation was conducted due to the precipitous manner in which Point Pedernales drops into the Pacific Ocean, and its general rocky nature. Analysis of the bathymetric data shows only depths of 18 and 12 ft in close proximity to the shore. The depth of 12 ft was arbitrarily established as the base of the rocky promontory

and the beginning of the sediment-covered slope. Results of these computations are shown below along with the longshore transport estimates of Bowen and Inman<sup>60</sup> for Surf, California, located 4 miles north of Point Pedernales.

<u>Longshore Transport Estimates for Point Pedernales (in <math>10^3</math> yd<sup>3</sup>/yr)</u>				
<u>Reference</u>	<u>Upcoast Transport</u>	<u>Downcoast Transport</u>	<u>Gross Transport</u>	<u>Net Transport Downcoast</u>
Bowen and Inman <sup>60</sup>	254	307	561	53
Model (all offshore zone transport)	305	519	824	214
Model (offshore zone transport in depths greater than 12 ft)	217	240	457	23

#### Discussion of the Results of Verification

43. The above tabulation shows that an approximate verification was achieved when the results of the computations were considered in relation to the assumptions made. Reliability of the data used for verification is somewhat unknown although it is certainly as good as or better than any other such data. The longshore transport estimates of Bowen and Inman<sup>60</sup> are based on calculations of the longshore component of wave power. Although this method seems to work out quite well in the overall sediment budget for this particular region of the California coast, it is not based on observed prototype measurements; it is, however, the best information available.

44. Computations made with the model, considering all longshore transport in the offshore zone, show consistently high results relative to the estimates of Bowen and Inman. This could be because no shallow-water cutoff point for sediment transport was established for this computation or it could be due to some of the assumptions inherent in the method of calculation. It should be reemphasized that the model assumes sediment is available for movement.

45. A much better estimate of the sediment transport is achieved

by restricting the transport to depths in excess of 12 ft; justification for this restriction is based on the existence of the rock promontory. Although smaller than the estimate of Bowen and Inman, the calculated values for northerly and southerly transport are similarly proportioned. Direction of the net drift is the same as for the estimate which includes movement in depths less than 12 ft; however, its relative magnitude is significantly reduced. This is primarily due to the fact that the waves coming from the northwest and north-northwest are of relatively low heights for most of the year, and will produce sediment movement only in shallower depths. Waves from the west-southwest and west are higher and produce sediment movement in deeper water. Therefore, by establishing a 12-ft cutoff point as the shallowest depth at which sediment can move around the promontory, waves from the west-southwest and west produce a greater relative effect on the estimate.

## PART VI: RECOMMENDATIONS

46. Recommendations are of two general categories: those of a general nature pertaining to the verification of the model and those that are more specific pertaining to modifications of the model.

- a. With few exceptions, most sediment transport models in the past have used previously existing wave data of unknown or questionable reliability or gathered wave data on a "make-do" basis. Therefore, it is recommended that a coordinated, well-planned program be undertaken to gather wave data. For example, a concerted effort must be made when designing and installing wave data instrumentation to ensure that it is capable of performing properly during periods of extremely severe wave action, since it is during these periods that the potential for sediment movement is greatest. Concurrently, detailed bathymetric surveys of the verification area should be performed. In the event of a storm severe enough to modify the morphology of the immediate area in a few days, the usual sediment tracing techniques might prove inadequate to provide information on the movement of such great amounts of sediment. One could then resort to a poststorm survey to obtain a gross measure of the magnitude of the movement. Moreover, it is important that the distribution of sediment sizes be determined in order to verify a model. This could be done in conjunction with a bathymetric survey and also would provide valuable information on the redistribution of sediment sizes in the event of a severe storm. An array of current meters should be installed. In addition to meters positioned to determine current velocity as a function of distance from shore, there should be at least one location near the breaker zone where the vertical velocity distribution is measured.
- b. A list of recommended modifications in order of their estimated overall importance that could be incorporated in the model is as follows:
  - (1) The capability of modeling suspended material as well as that in the bed load should be examined (cf. MacDonald<sup>46</sup>).
  - (2) Effects of using a specified distribution of sediment sizes rather than assuming a descriptive mean grain size should be evaluated. In addition, an attempt should be made to determine the feasibility of extending the range of sediment sizes to which the model can be applied.

- (3) Accuracy might be improved by modification of the assumed forms of the velocity distributions in both the laminar and turbulent boundary layers (cf. Sleath<sup>55</sup>).
- (4) A further evaluation of the applicability of the bed-load function over a wide range of conditions would be desirable.
- (5) The importance of using nonlinear wave theory to calculate the increase in wave height due to shoaling should be evaluated (cf. James<sup>70,71</sup>).

## PART VII: CONCLUDING REMARKS

47. The model as formulated is limited to transport seaward of the breaker zone and has a number of simplifying assumptions and limitations. It should be potentially valuable as a state-of-the-art engineering tool to assist in the placement of beach nourishment material. However, a comprehensive field verification should be performed prior to general application. The importance of this verification cannot be over-emphasized; a comprehensive and controlled data collection program could serve to evaluate, not only the applicability of this model but future ones as well, thereby permitting potential future cost reductions for this type of study. The principal problem encountered in attempting to evaluate applicability of sediment transport models is the lack of reliable prototype data for verification.

48. The tabulation on page 45 shows estimates of longshore sediment transport at Point Pedernales, California. The absolute estimates of the longshore transport predicted by the model (for depths greater than 12 ft) are about 15 to 22 percent less than those of Bowen and Inman<sup>60</sup> and the ratios of the upcoast/downcoast transport are quite similar (0.83 and 0.90). This implies that while further refinement and calibration may be desirable, the physics of the model is fundamentally sound.

49. Figure 10 shows the gross annual sediment transport predicted by the model as a function of distance from shore and water depth; no significant longshore transport occurs in depths greater than 50 ft. The dashed portion of the curve indicates the zone where wave breaking occurred; therefore, all or part of this region was the surf zone depending upon the existing wave condition. Figure 11 shows the net annual sediment transport predicted by the model, again as a function of distance from shore and water depth. Note that at a depth of about 20 ft, there is no net upcoast/downcoast movement indicated and that on an annually averaged basis, material deposited at depths of 20 ft or less can be expected to move downcoast, while material deposited at depths of 20 to 50 ft can be expected to move upcoast.

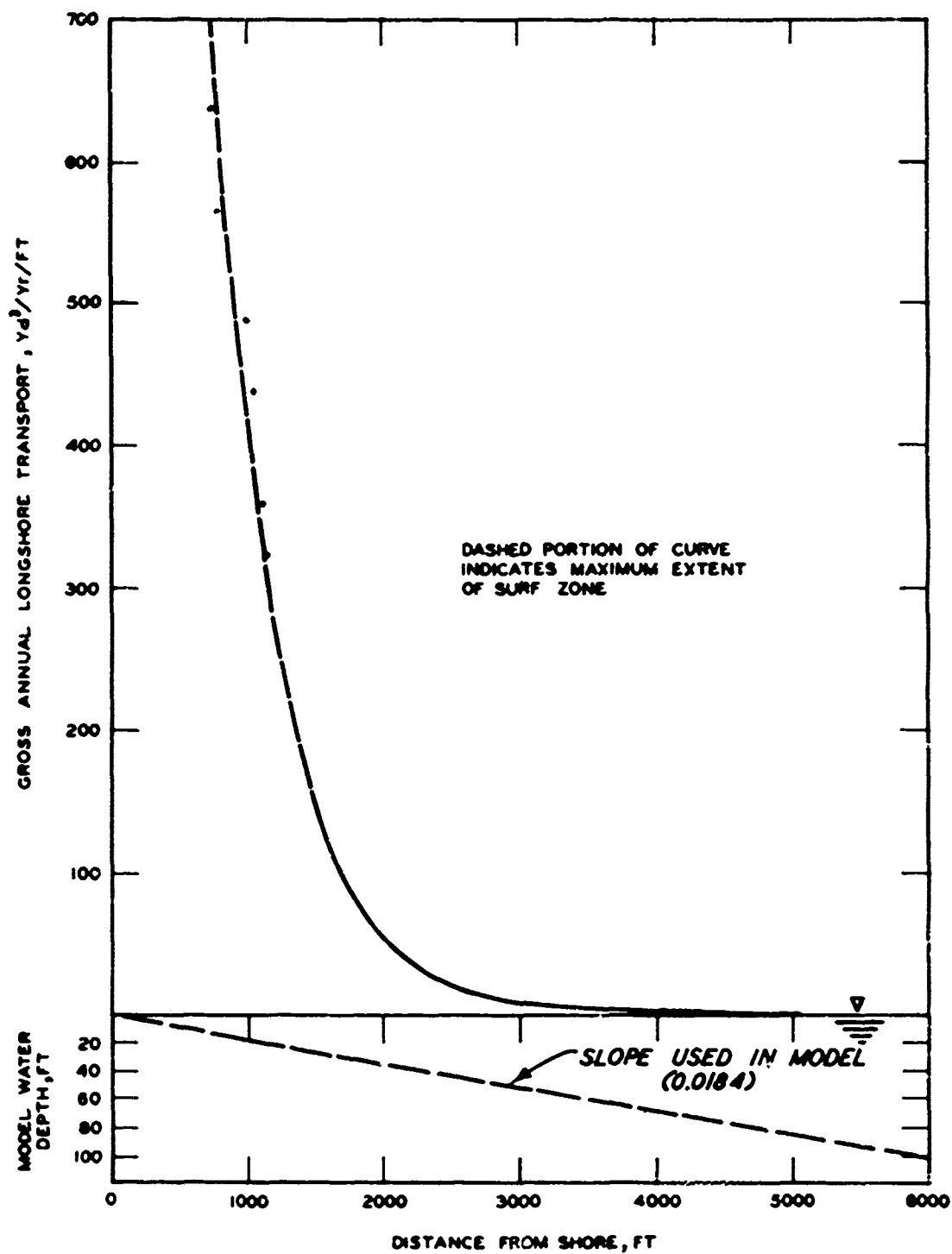


Figure 10. Gross annual longshore transport as a function of distance from shore, Point Pedernales, California

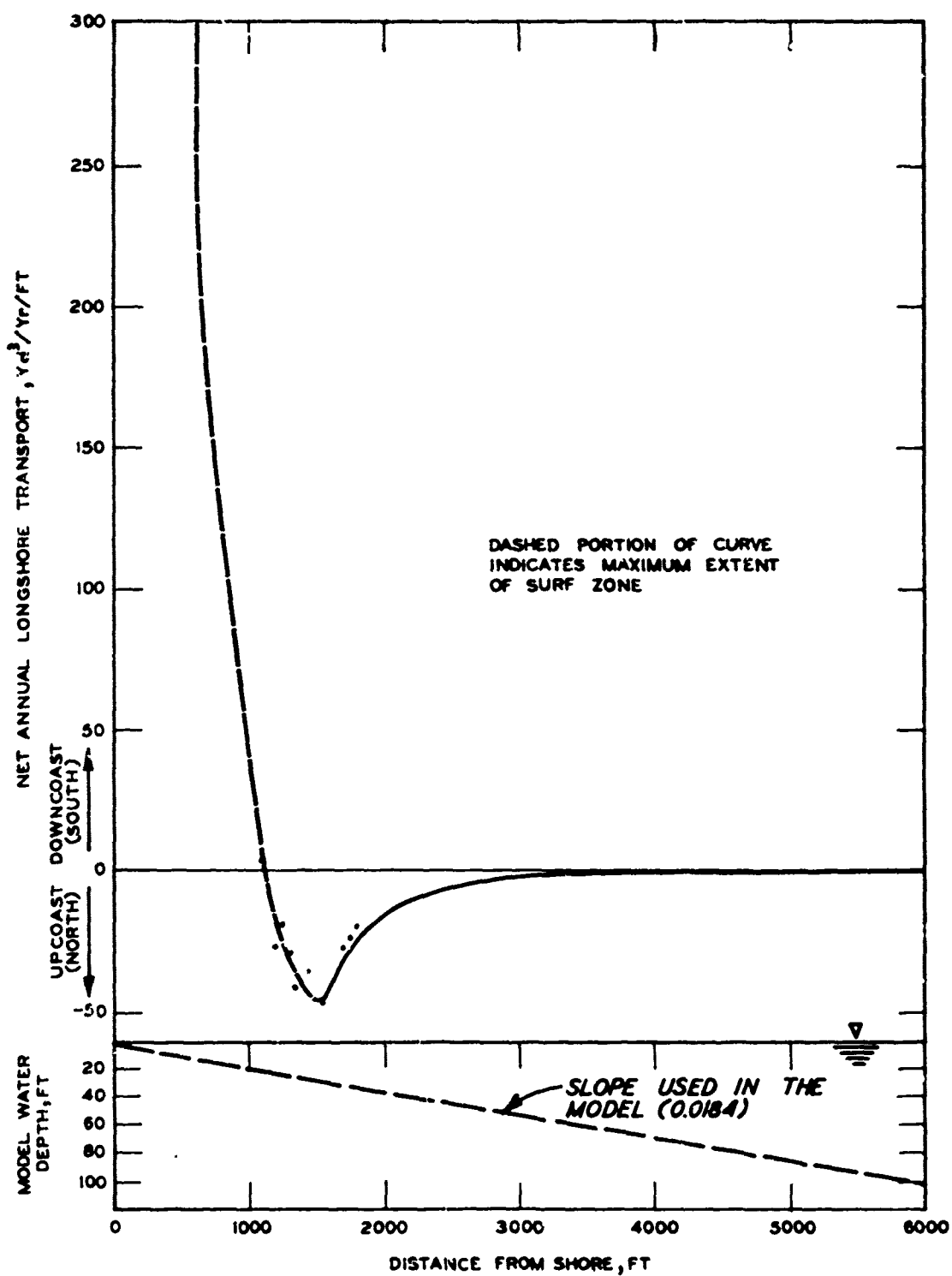


Figure 11. Net annual longshore transport as a function of distance from shore, Point Pedernales, California



50. Because the wave climate determines the amount of energy available to initiate sediment movement along any given stretch of beach, the amount of sediment moved at a given location is dependent upon the wave climate at that location. In other words, the wave energy available to move sediment is not propagated in the longshore direction. An eroding beach, therefore, can be aided by depositing suitable material offshore at a water depth where it will be entrained (and thus sacrificed) rather than depositing the material directly on the beach and having it eroded. There is the possibility, of course, that the deposited material will be moved onshore opposite its placed location but to determine this quantitatively in a general location appears beyond the present state of the art.

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# APPENDIX A: NOTATION

a	Amplitude of horizontal displacement of water particle at bottom due to wave action
A,B	Empirically derived constants where best fit the theoretical curve to the experimental data of Kalkanis and Abou-Seida
$\langle B_y \rangle$	Mean stress on bottom
$C_B$	Average bed-load concentration
$C_n$	Wave group velocity
$D_{50}$	Mean sediment diameter; $2D_{50}$ , thickness of bed layer
E	Wave energy density
$f_1(z)$	Velocity amplitude function
$f_2(z)$	Phase shift function
g	Acceleration due to gravity
h	Water depth below still-water level
H	Wave height
$I_e$	Rate of transport of immersed weight of sand past section of beach
k	Wave number
K	Von Karman's constant
K'	Dimensionless factor of proportionality
m	A random variable that has a normal distribution of zero mean and a standard deviation of one
M	Coefficient of proportionality
n	Integer determined by the magnitude of the Reynolds number
N	Nondimensional constant; coefficient of proportionality
P	Horizontal mixing coefficient
$q_B$	Oscillatory bed-load rate
$Q_B$	Net bed-load concentration
R	Reynolds number
s	Slope
S	Friction coefficient
t	Time
T	Wave period
$u_o$	Parameter
U	Horizontal velocity
$U_m$	Maximum orbital velocity of waves



$U_T$	Velocity within the boundary layer
$U(z)$	Depth-dependent horizontal velocity
$U_\infty$	Horizontal fluid velocity outside boundary layer
$v$	Fluid velocity
$v_0$	Maximum horizontal bottom velocity outside boundary layer
$V, X$	Nondimensional variables
$V_l$	Longshore velocity
$\bar{v}_m$	Average velocity at a distance $D_{50}$ from theoretical bed
$\bar{v}_u$	Mean unsteady velocity within boundary layer
$x$	Horizontal distance normal to shoreline
$y$	Horizontal distance parallel to shoreline
$z$	Vertical distance positive downward
$\alpha$	Angle of wave crest with respect to shoreline; function of $x$
$\alpha_b$	Angle of wave crest at breaking with respect to shoreline
$\beta$	Wave breaking coefficient
$\beta_0$	Reciprocal of boundary layer thickness
$\gamma_s$	Unit dry weight of bed material
$\delta$	Thickness of the boundary layer
$\delta^*$	Laminar sublayer thickness
$\epsilon$	Roughness diameter
$\eta_0$	Empirically derived constant which best fits the theoretical curve to the experimental data of Kalkanis and Abou-Seida
$\theta$	Phase angle
$\mu$	Dynamic viscosity
$\mu_e$	Coefficient of horizontal eddy viscosity
$\nu$	Kinematic viscosity
$\xi$	Hiding factor
$\Xi$	Function of $x$
$\rho_f$	Density of fluid
$\rho_s$	Density of sediment
$\rho_w$	Density of seawater
$\tau_y$	Net stress in $y$ direction
$\phi$	Angle
$\Phi$	Bed-load function
$\psi$	Flow intensity function
$\omega$	Wave frequency